Dynamical Effects of Accelerometer Misalignment Assumptions on Gravity Recovery and Climate Experiment

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Nomenclature

a	linear acceleration, m/s^2
b, b', c, c', d, d'	indeterminate constants
U	gravitational potential
x, y, z	inertial frame position vectors
X, Y, Z	body frame position vectors
α	angular acceleration, rad/s^2
Δv	change of a vector v
$\mathcal{H}(f)$	Hessian of a function f
∇f	gradiant of a function f
ω	angular velocity, rad/s
ν	phase of an elliptical orbit;
$\|\|w\|\|$	(vector) norm of w
Subscripts	
ACC	from accelerometer measurements
gg	from gravity gradient
ng	non-gravitational
n	in normal direction
r	in radial direction
t	in tangential direction

I. Introduction

Due to both technological constraints on the construction and positioning of the accelerometer as well as other internal spacecraft dynamics (e.g. fuel levels and other dynamics), a misalignment almost always exists between the spacecraft's actual center of mass and the location of the accelerometers.[1]

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This means that the accelerometer readings are not exactly a faithful representation of the actual spacecraft center of mass dynamics. Usually, one does not model this discrepancy directly for orbit determination purposes. Instead it is treated as noise and an appropriate fitting will hopefully avoid any problems, for the differences are relatively small.

However, it is still interesting to know if there is any dynamical significance should we choose to model this deviation. Results of such estimates would then help us in make better engineering decision on choosing the adaquate accelerometer setup as well as reassure our confidence in the tracking and orbit determination results. Hence we analyzed the dynamical effects of modeling these deviations for the Gravity Recovery and Climate Experiment (GRACE) in this study.

II. Dynamical Models

II.A. Analytic Analysis

We have the following governing equations of motion

$$\ddot{x} = \nabla U(x) + a_{ng} \tag{1}$$

$$y = \mathbf{R}(t) Y \tag{2}$$

$$z = \mathbf{R}(t) Z \tag{3}$$

Here x, X represent the geocentric center of mass position vector of our spacecraft (in inertial and body-fixed frames, resp.), y, Y the vector from center of mass to on-board accelerometer, and z, Z the vector from center of mass to antenna phase center. U is the gravitational potential and $\mathbf{R}(t)$ the frame transformation from the body-fixed frame to the inertial reference frame. See Figure 1 below.

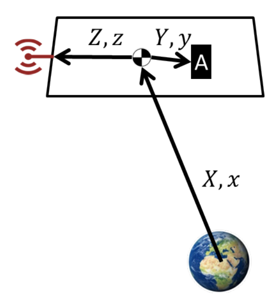

Figure 1. Graphical representation of the system.

The goal is to study how does the position offset of the accelerometer from the center of mass (a nonzero Y) affect our knowledge of spacecraft dynamics. Since we have range measurements of the phase center from tracking, we want to compare its measured position with the propagated value of $x + z$ under different models of Y.

If we assume Y to be constant for simplicity, we will additionally have

$$\begin{aligned}\dot{y} =&\dot{\mathbf{R}}Y + \mathbf{R}\dot{Y} = \dot{\mathbf{R}}Y = \omega \times y \\ \ddot{y} =&\omega \times \dot{y} + \dot{\omega} \times y = \omega \times (\omega \times y) + \alpha \times y \end{aligned} \tag{4}$$

Since the reference frame transformation $\mathbf{R}(t)$ are a fixed time series derived from physical measurements[a], comparing x is good enough as long as Z stays constant. Then the only term left to be expanded is the non-gravitational acceleration a_{ng} in Equation 1. Recall from the measurement equation,[1] we have

$$a_{ng} = -a_{ACC} - \ddot{y} + a_{gg} \tag{5}$$

where a_{ACC} are the accelerometer measurements and a_{gg} the gravity gradient term, which is simply

$$\begin{aligned} a_{gg} =&\nabla U(x + y) - \nabla U(x) \\ \approx&\mathcal{H}(U(x))y \end{aligned} \tag{6}$$

neglecting the $\mathcal{O}(|y|^2)$ terms[b]. Here $\mathcal{H}(U(x))$ is the Hessian of the gravitational potential at x. Now if we substitute Equations 4, 5, and 6 back into $\ddot{x}$ (Equation 1), it is clear that we can factor out a scalar $||Y||$ as all terms are either not a function of Y or linear with respect to Y.

II.B. Order of Magnitude Estimates

To gauge how significant and sensible that our consideration of the dynamical effects from a reasonable deviation between the accelerometer and spacecraft center of mass are, we used GRACE telemetry data[2c], assuming $||Y||$ to be $300\mu m$, $||Z||$ to be $1.5m$[d], and the scalar and J2 terms of the gravitational potential U to obtain estimates of $||a_{gg}||$ and $||\ddot{y}||$ to be approximately $7nm/s$ and $0.8nm/s^2$, resp. Naive integration shows that these will yield deviations in x on the order of 10^2 meters over one day.

II.C. Parametric Study

Based on the above assumptions and analysis, the only varying parameter is a unit vector in $\mathbb{R}^3$ accounting for the direction of accelerometer misalignment since we can factor out the scalar $||Y||$[e].

Given this input, we can propagate two trajectories of the spacecraft with and without the misalignment. The difference trajectory Δx in Radial-Tangential-Normal (RTN) frame can then be modeled in two meaningful ways. The first (**Model A**) takes into account only the differences in the initial conditions between the two orbits[f]:

$$\Delta x_r = b_r + c_r \cos(\nu) + d_r \sin(\nu) \tag{7}$$

$$\Delta x_t = b_t + b_t' \Delta T + c_t \cos(\nu) + d_t \sin(\nu) \tag{8}$$

$$\Delta x_n = b_n + c_n \cos(\nu) + d_n \sin(\nu) \tag{9}$$

where all the coefficients are some indeterminate constants, and ν is the phase of the orbit, i.e. the sum of its argument of periapsis and its true anomaly.

If we instead also take into account the effects of the ballistic coefficient (i.e. effects of the derivative of the mean motion) to model drag, we obtain a second model (**Model B**)[g]:

$$\begin{aligned}\Delta x_r =&b_r + (c_r + c_r' \Delta T) \cos(\nu) \\ &+ (d_r + d_r' \Delta T) \sin(\nu) \end{aligned} \tag{10}$$

$$\begin{aligned}\Delta x_t =&b_t + b_t' \Delta T + b_t' \Delta T^2 + (c_t + c_t' \Delta T) \cos(\nu) + \\ &(d_t + d_t' \Delta T) \sin(\nu) \end{aligned} \tag{11}$$

$$\begin{aligned}\Delta x_n =&b_n + b_t' \Delta T + (c_n + c_n' \Delta T) \cos(\nu) + \\ &(d_n + d_n' \Delta T) \sin(\nu) \end{aligned} \tag{12}$$

where all the coefficients are again constants and ν still the phase of the orbit.

With these models, we can simply perform a generic least-squares analysis and study the relationship between the root-mean-square (RMS) of the post-fit residues versus the given input unit vector.

III. Results and Discussion

III.A. Numerical Integrations

We used actual state of GRACE spacecraft A on 2008/05/18 00:00:00 (UTC) as initial conditions, attitude data for angular velocity[h], and accelerometer readings for both $a_{ACC}(t)$ and $\alpha(t)$, cf. GNV1B, SCA1B, and ACC1B in [Case et. al.][2]

[a] GRACE satellites have on-board star cameras to calibrate the attitudes. See Section III.A, footnote h, and SCA1B in [Case et. al.][2]

[b] This is warranted as we would be dealing with things below ordinary precision. Also see [Milani and Gronchi][1]

[c] All accelerometer readings used have daily averaged biases removed.

[d] These numbers are derived from the mission specification and geometrical configuration of the satellite[3]

[e] In practice, it should be rather obvious that minimizing $||Y||$ is the most desirable thing to do. Hence its linearity (see end of Section II.A) means that we won't reach any more interesting conclusions by varying it.

[f] This corresponds to the homogenous solution to the Hills equations, cf. Equation 1.13 in [Coloumbo][4]

[g] This corresponds to the oscillatory solution of the Hills equations, cf. Equation 1.17 in [Coloumbo][4]

[h] It was derived from the quaternion representation $q(t)$ of $\mathbf{R}(t)$ using the following interpolation

$$\omega(t) = \frac{2}{\Delta t} ln(q(t + \Delta t) \otimes q^{-1}(t)) \tag{13}$$

Integrating with the Adams-Bashforth method under quadruple precision[i], we get the following plots of post-fit residues with respect to different input misalignment directions in Figure 2 and 3. Here the directions are expressed in standard longitude and latitude coordinates in the Science Reference Frame (SRF[j]), and they lie on a rectangular grid of $3° \times 3°$ resolution. Specifically, the north edge of the graph represent the radial direction away from the Earth, while the center of the graph points along track (towards the other GRACE spacecraft B).

Under **Model A**, the RMS of residues seem to have simple modes. In the radial and tangential directions they peak near $-120°$ and $-60°$ on the central plane normal to the radial direction and passing through the center of mass. In the normal direction, they have local minima around $-30°$ and $150°$ on the same plane.

For **Model B**, more complicated behaviors emerge as sharper fits are achieved. In the radial direction, there are peaks near $(-120°, 15°)$ and $(60°, 0°)$ plus minima near $(-30°, -30°)$ and $(150°, 30°)$ in SRF (longitude, latitude) coordinate. For the tangential direction, the peaks are near $(-90°, 30°)$ and $(90°, -30°)$ while the minima are near $(-120°, -60°)$ and $(60°, 60°)$. In the normal direction, we only get minima near $(165°, -45°)$ and $(-15°, 45°)$.

Since we eliminated internal satellite dynamics by assuming constant Y and Z, these results should be dominantly driven by the gravity gradient component and the (angular) acceleration profile from GRACE on-board telemetry.

III.B. Further Studies

Although other spacecrafts in low-earth orbits with angular acceleration profile similar to GRACE should experience similar effects demonstrated in this study, it would be interesting to see if and how other angular acceleration profiles will influence the results.

More sophisticated modeling on the misalignment could also be developed. Most significantly is probably to relax the assumption that Y and Z are constant, in order to model internal spacecraft dynamics. Other factors such as the altitude of the orbit could also be considered as an additional parameter for a more general study, then the findings can be more readily applied to future missions.

[i]This is to ensure our results are free of numerical artifacts.
[j]See GRACE product specification document[3] pp. 16-18

IV. Conclusions

We have shown that there is a significant difference in predicting GRACE spacecraft orbit dynamics whether we choose to model the misalignment between the on-board accelerometer and the center of mass of the spacecraft or not. Even assuming constant misalignment in the body frame, we showed that the gravity gradient effects and (angular) acceleration of the spacecraft can still generate some interesting deviations as a function of varying direction of the misalignment vector Y.

Acknowledgments

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References

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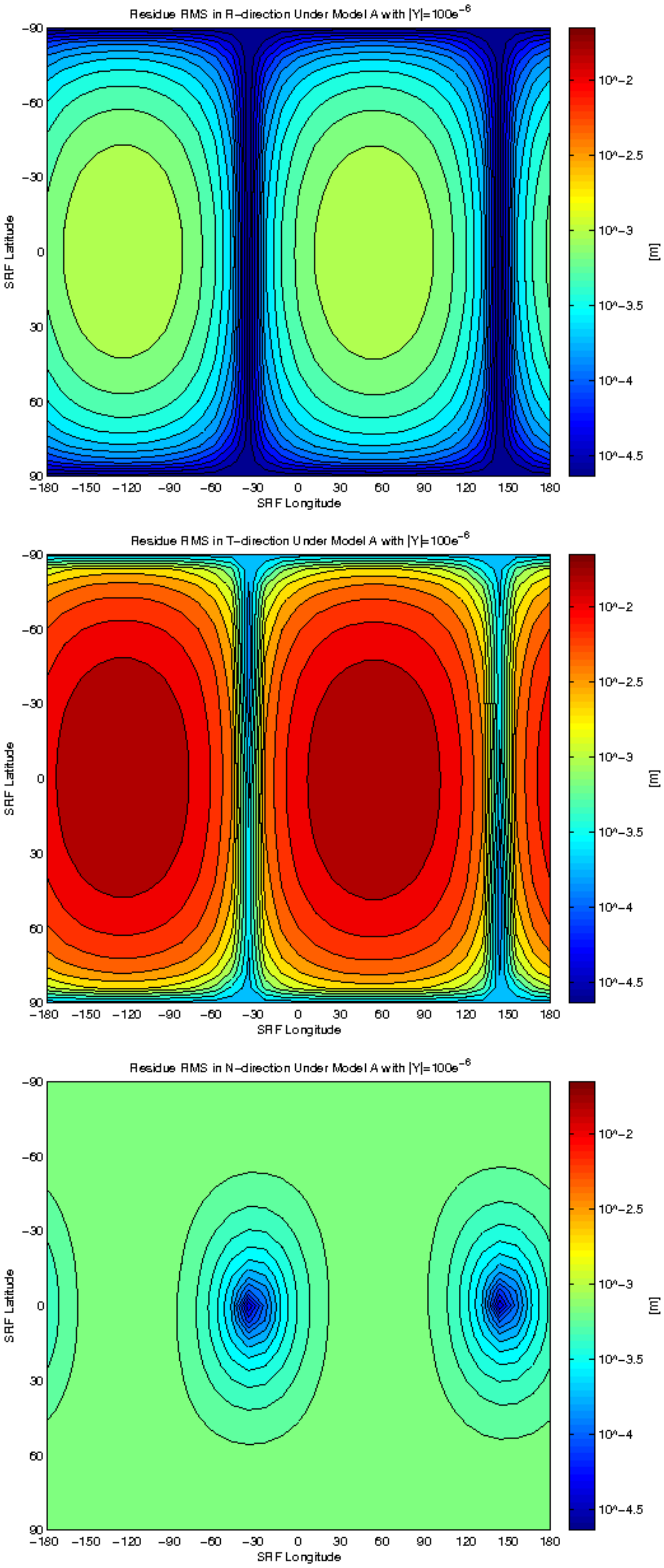

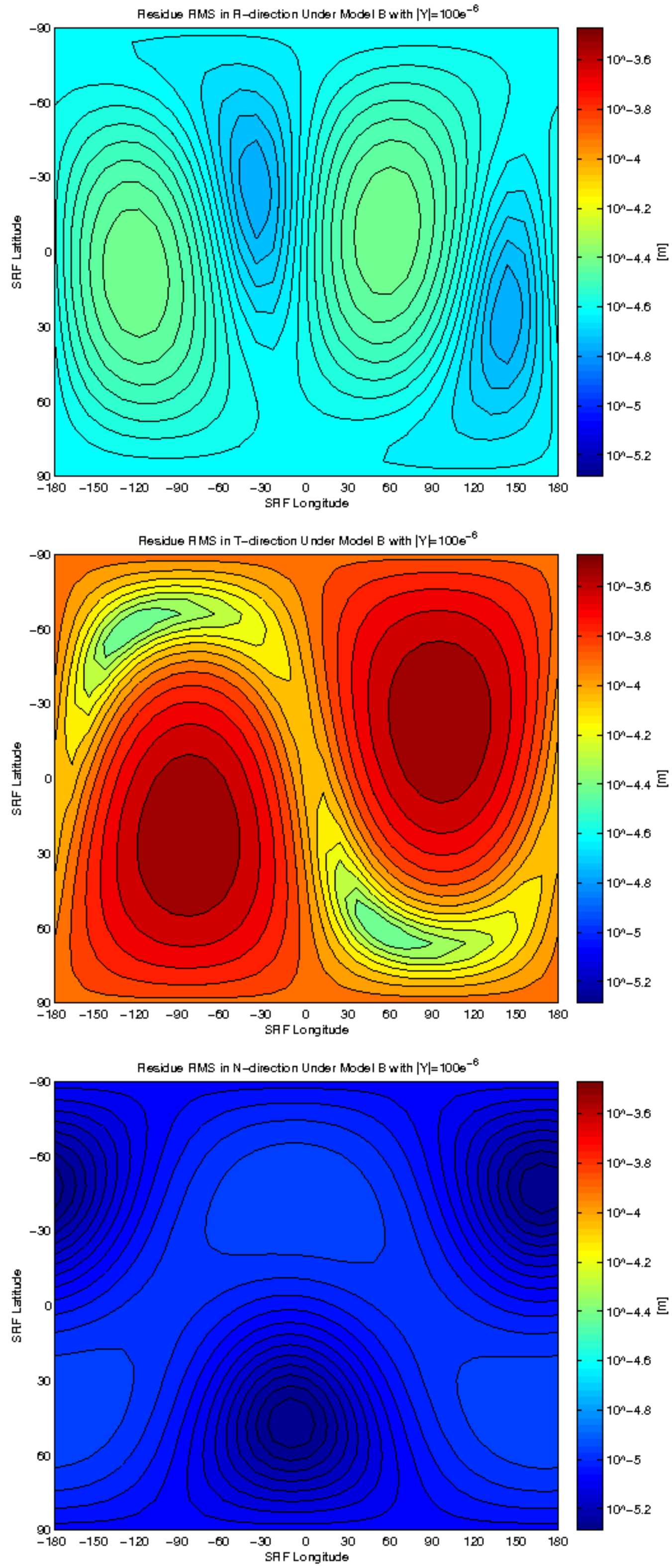

Figure 2. RMS residues [m] in R-, T-, N-directions (from top to bottom) under Model A. All results assumed $||Y|| = 100[\mu m]$ without loss of generality.

Figure 3. RMS residues [m] in R-, T-, N-directions (from top to bottom) under Model B. All results assumed $||Y|| = 100[\mu m]$ without loss of generality.