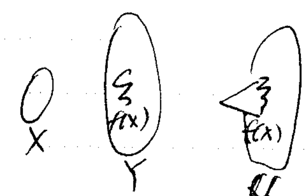
goal of my talk is to briefly explain what cofiber & fiber sequences are. & to use them to show the awesome fact that there is a "3-dim hole" inside a 2-sphere.

so for this objective, we are trying to understand (topological) spaces.
One obvious way to study something unknown is to ~~learn~~ study the ~~about properties of~~ maps into & out of spaces that we know about. the natural problem of this approach is that. a lot ~~most~~ of the time, maps behave badly.
And the aim of cofiber & fiber sequences is to study how similar is our arbitrary maps between topological spaces are infact a "nice" injection. and/or a "nice" surjection.

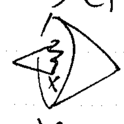
Now given top sp. X, Y & $f: X \to Y$. we have the homotopy cofiber Cf to form a chain.

$X \xrightarrow{f} Y \xrightarrow{i} Cf$. pictorially.



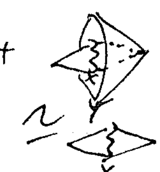
idea is to contract/quotient out image of X. w/o disturbing the internal structure of Y.

what's interesting about this construct. is that. we can continue this seq. indefinitely.
e.g. the 2nd step is $Y \xrightarrow{i} Cf \to Ci$.
pictorially Ci is



since i is inclusion map.

b/c we are contracting the copy of Y, we can deduce that



we call this thing the $\underline{\Sigma X}$. in fact ~~we~~ I claim that we can $\Sigma X \to \Sigma Y \to \Sigma Cf \to \Sigma^2 X \to \dots$
Σ is a very nice map. infact an endofunctor.
the key fact to remember is $\Sigma S^n = S^{n+1}$ look at picture

another cool fact is that it admits a nice adjoint functor Ω. $\underline{\Omega X}$ is space of loops in X.
explicitly the adjunction enforces a homeomorphism $Maps(\Sigma X, Y) \cong Maps(X, \Omega Y)$
It is bit hard to see a picture (although not completely hopeless.)
but this adjunction give rise to the dual notion of fiber sequence. by reversing all the arrows as

$\dots \to \Omega^2 Y \to \Omega Ff \to \Omega X \to \Omega Y \to Ff \xrightarrow{\pi} X \xrightarrow{f} Y$

this contruct in fact measures how close our map f is to a "nice" surjection

TensorFlow

Theano
Autograd.
Keras.

now we have ~~the~~ following special map

$U_1 \to SO_3 \to S^2$ ~~$\to SO_3$~~

$S^3 \xrightarrow{f} S^2$ called "the Hopf bundle".

~~And it can be shown~~.

$$\begin{array}{ccccc} & & X & & Y \\ & & \| & & \| \\ Ff & \to & S^3 & \xrightarrow{f} & S^2 \\ \wr\| & & & & \\ S^1. & & & & \end{array}$$

And there are many ways to show that in fact

for example using quaternions.

this means that we can form the corresponding fiber sequences. as before.

(Special case of) Thm: for any fiber sequence, we can obtain a long exact sequence of abelian groups.

$$\to [S^2, \Omega Ff] \to [S^2, \Omega X] \to [S^2, \Omega Y] \to [S^2, Ff] \to [S^2, X] \to [S^2, Y].$$

Here $[X, Y] := \mathrm{Maps}(X, Y)/\text{homotopy}$. ← what this means is two maps are equiv. if we can cts deform to other.

~~for two~~ In this setting, exactness means exactly what you expect, namely kernel of a map is ~~exactly~~ precisely the image of the preceding map.

now we plug in the Hopf map & use the facts about the adjunction between Ω & Σ. & $\Sigma S^n = S^{n+1}$.

$$\cdots \to [\Sigma S^2, S^1] \to [\Sigma S^2, S^3] \to [\Sigma S^2, S^2] \to [S^2, S^1] \to \cdots$$
$$\Sigma S^2 = S^3 \qquad \Sigma S^2 = S^3 \qquad \Sigma S^2 = S^3.$$

Facts: ① $[S^m, S^m] \cong \mathbb{Z}$. realized via the degree map. as one can wrap S^n around any integer # of times

② $[S^k, S^1] \cong 0 \quad \forall k \neq 1$. this is b/c $\mathbb{R}$ is universal cover of S^1. via ~~...~~ $t \mapsto \exp(2\pi i t)$

& covering space theory ~~which~~ tells us that map to S^1 can be lifted to map to $\mathbb{R}$, which is contractible, which must be continuously deformable to the trivial constant map.

so $$0 \to \mathbb{Z} \to [S^3, S^2] \to 0.$$

exactness $\Rightarrow [S^3, S^2] \cong \mathbb{Z}$

$\|$

in other notations $\pi_3(S^2)$.

we have an entire family of maps from S^3 to S^2.

One say we have a wierd "3-dim hole" in 2-sphere.