Effects of Accelerometer misalignment with S/C CM

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- Theoretical Estimates
- Integrated Solutions and LS Fits
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Assumed Eqns. Of Motion

- Small letters are in ECI frame, capital letters in body frame
- x is the vector from ECI origin to S/C cm

$$\ddot{x} = \nabla U + a_{ng}$$

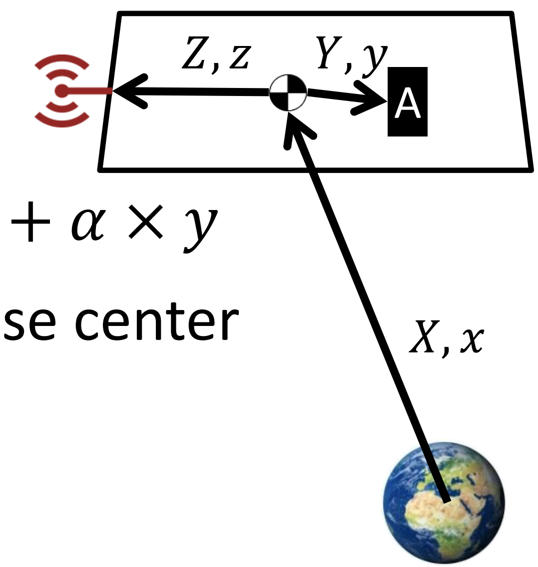
- y is the vector from S/C cm to accelerometer

$$y = \mathbf{R}Y$$
$$\dot{y} = \dot{\mathbf{R}}Y + \mathbf{R}\dot{Y} = \dot{\mathbf{R}}Y = \omega \times y$$
$$\ddot{y} = \omega \times \dot{y} + \dot{\omega} \times y = \omega \times (\omega \times y) + \alpha \times y$$

- z is the vector from s/c cm to antenna phase center

$$z = \mathbf{R}Z$$

- Y, Z are assumed to be constant



Non-gravitational accelerations

- Measurement equation

$$a_{ng} = -a_{ACC} - \ddot{y} + a_{gg}(y)$$

- Gravity gradient, neglecting $\mathcal{O}(|y|^2)$

$$a_{gg} = \nabla U(x + y) - \nabla U(x)$$

$$a_{gg} \approx \begin{bmatrix} U_{xx} & U_{xy} & U_{zx} \\ U_{xy} & U_{yy} & U_{yz} \\ U_{zx} & U_{yz} & U_{zz} \end{bmatrix} y$$

Gravitational Potential

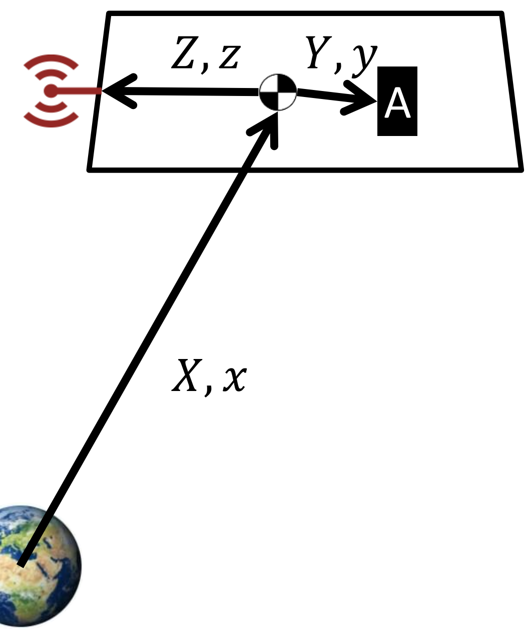
- Taking into account of the scalar and J2 component only, we have

$$U(x) = \frac{\mu}{\|x\|} + \frac{\mu R_E^2 J_2}{\|x\|^3}\left(\frac{3}{2}\sin^2\phi - \frac{1}{2}\right)$$

$$= \frac{\mu}{\|x\|} + \frac{3}{2}\mu R_E^2 J_2 (\frac{x_z^2}{\|x\|^5} - \frac{1}{3\|x\|^3})$$

Problem Statement

- We want to study how does the position offset of the ACC from CM (a nonzero Y) affect our knowledge of s/c dynamics
- Specifically, since we have range measurements of the phase center from tracking, we want to compare its measured position and propogated position $x + z$ under different models of Y
- But with Z assumed constant, and fixing the frame transformation from data, comparing x is good enough



Theoretical Estimates

- We assume

$$\|Y\| \sim 300\ \mu m$$
$$Z \sim [1.5\ 0\ 0]^T\ m$$

- From data (daily averaged bias removed for accelerations)

$$|\omega_x| \lesssim 20, \qquad |\omega_y + .0011| \lesssim 50, \qquad |\omega_z| \lesssim 60\ \mu rad/s$$
$$|\alpha_x| \lesssim 1, \qquad |\alpha_y| \lesssim 0.7, \qquad |\alpha_z| \lesssim 0.2\ \mu rad/s^2$$
$$|a_{ACC_x}| \lesssim 40, \qquad |a_{ACC_y}| \lesssim 20, \qquad |a_{ACC_z}| \lesssim 60 nm/s^2$$

- Derived estimates:

$$\frac{\mu}{\|x\|^3} \sim 1.2E^{-6}, \qquad \frac{3\mu R_E^2 J_2}{2\|x\|^5} \sim 1.6E^{-9}, \qquad \|a_{gg}\| \sim 7 nm/s^2$$
$$\|\omega \times (\omega \times y)\| \lesssim 4E^{-10}, \|\alpha \times y\| \lesssim 4\mathrm{E}^{-10}, \|\ddot{y}\| \lesssim 0.8 nm/s^2$$

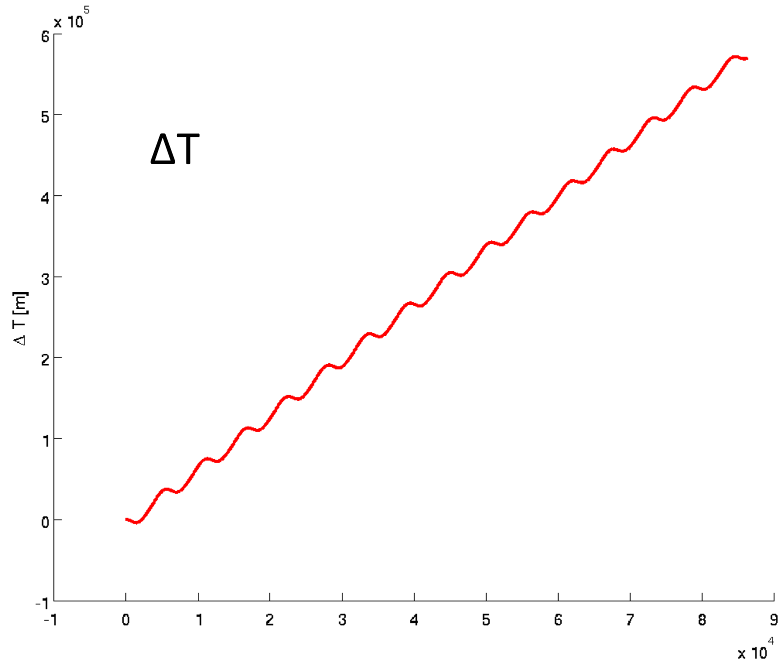
Integrated Solutions

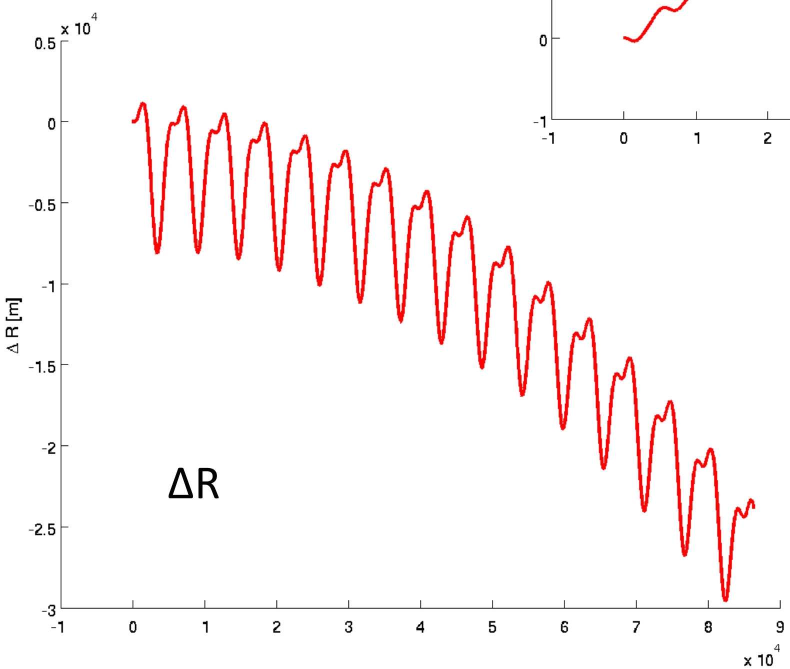
- Angular velocity obtained from quaternion presentation of $\mathbf{R}$

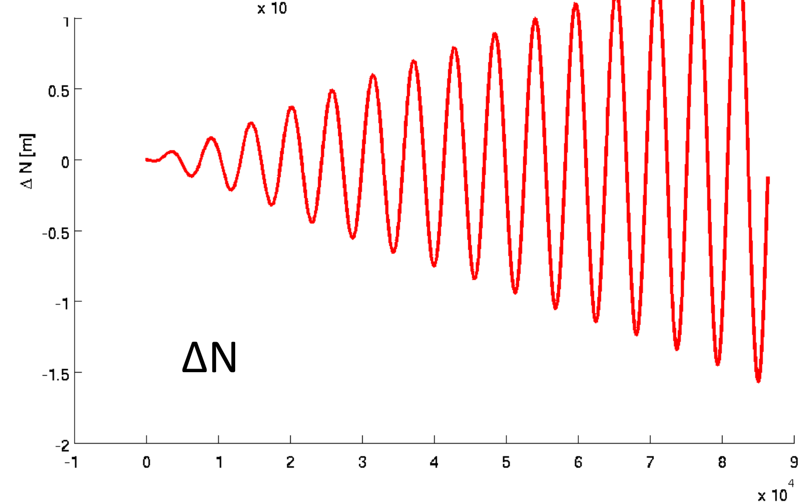
$$\omega(t) = \frac{2}{\Delta t} \ln\left(q(t+\Delta t) \otimes q^{-1}(t)\right)$$

- Y assumed to have the worst case magnitude of $300\ \mu m$ in radial (R), along-track (T), and cross-track (N) directions
- Initial condition obtained from GNV1B
- Integrating on day 2008-05-18 with quadruple precision using Adams-Bashforth method

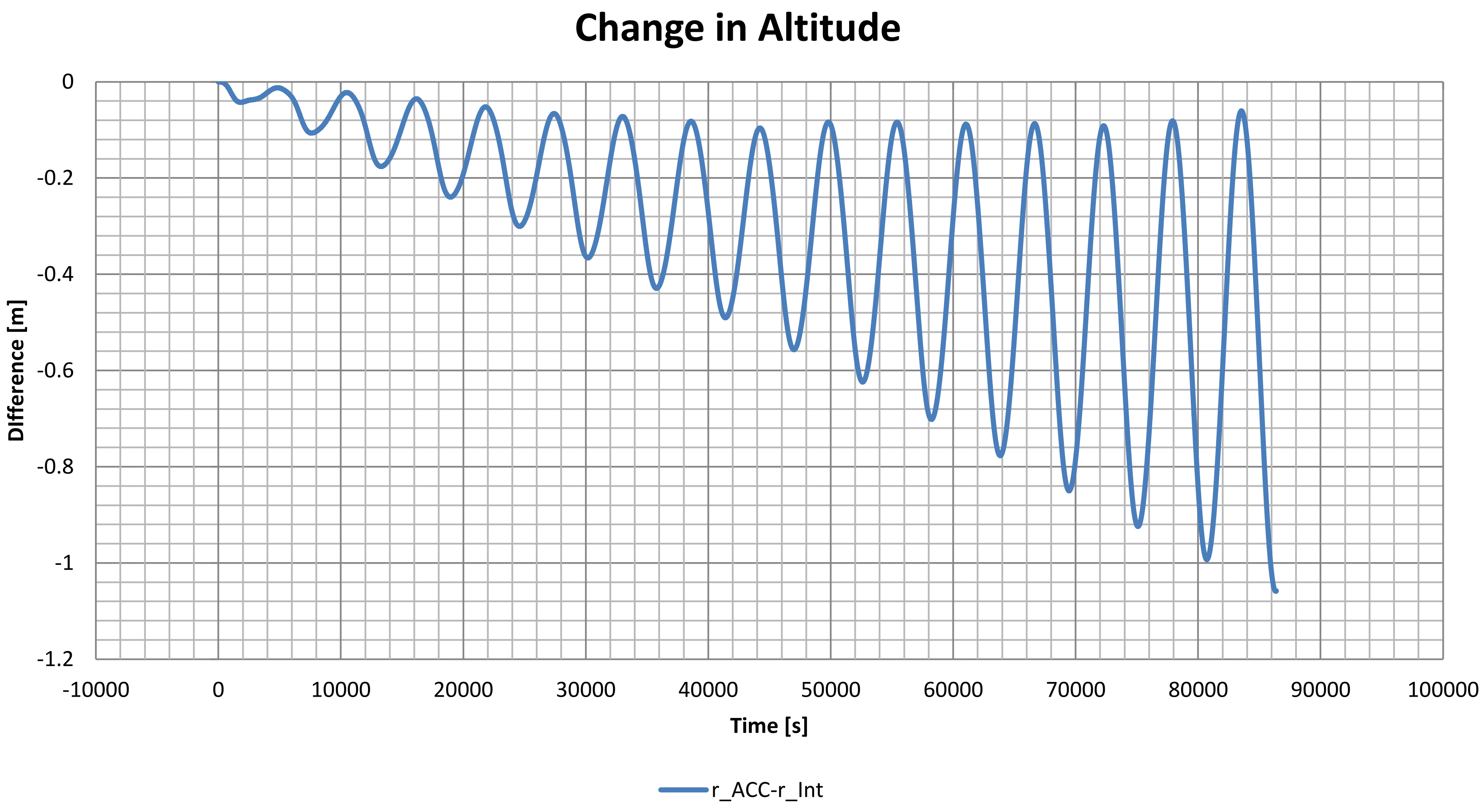
Results: Analytical vs. GNV1B



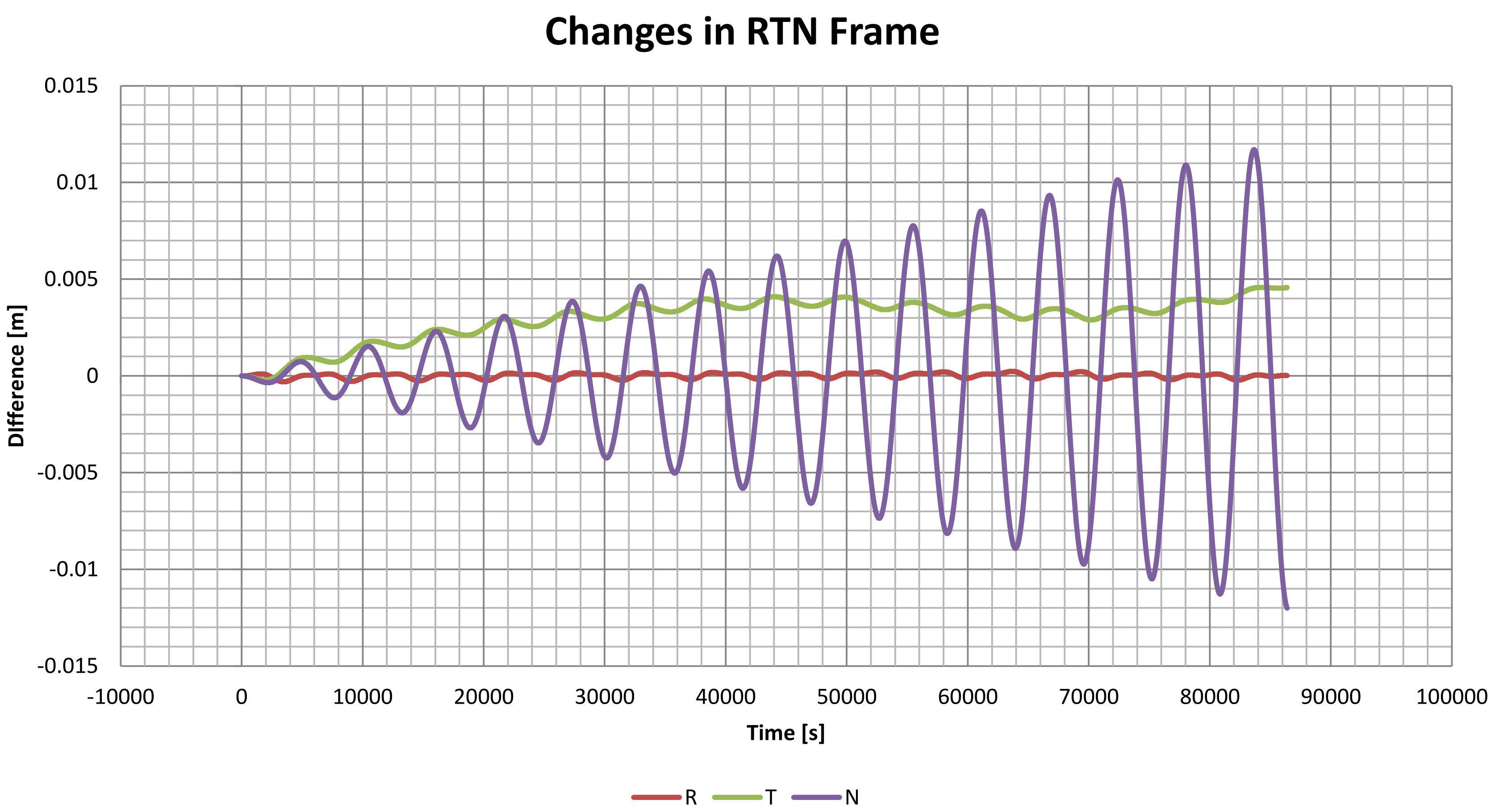




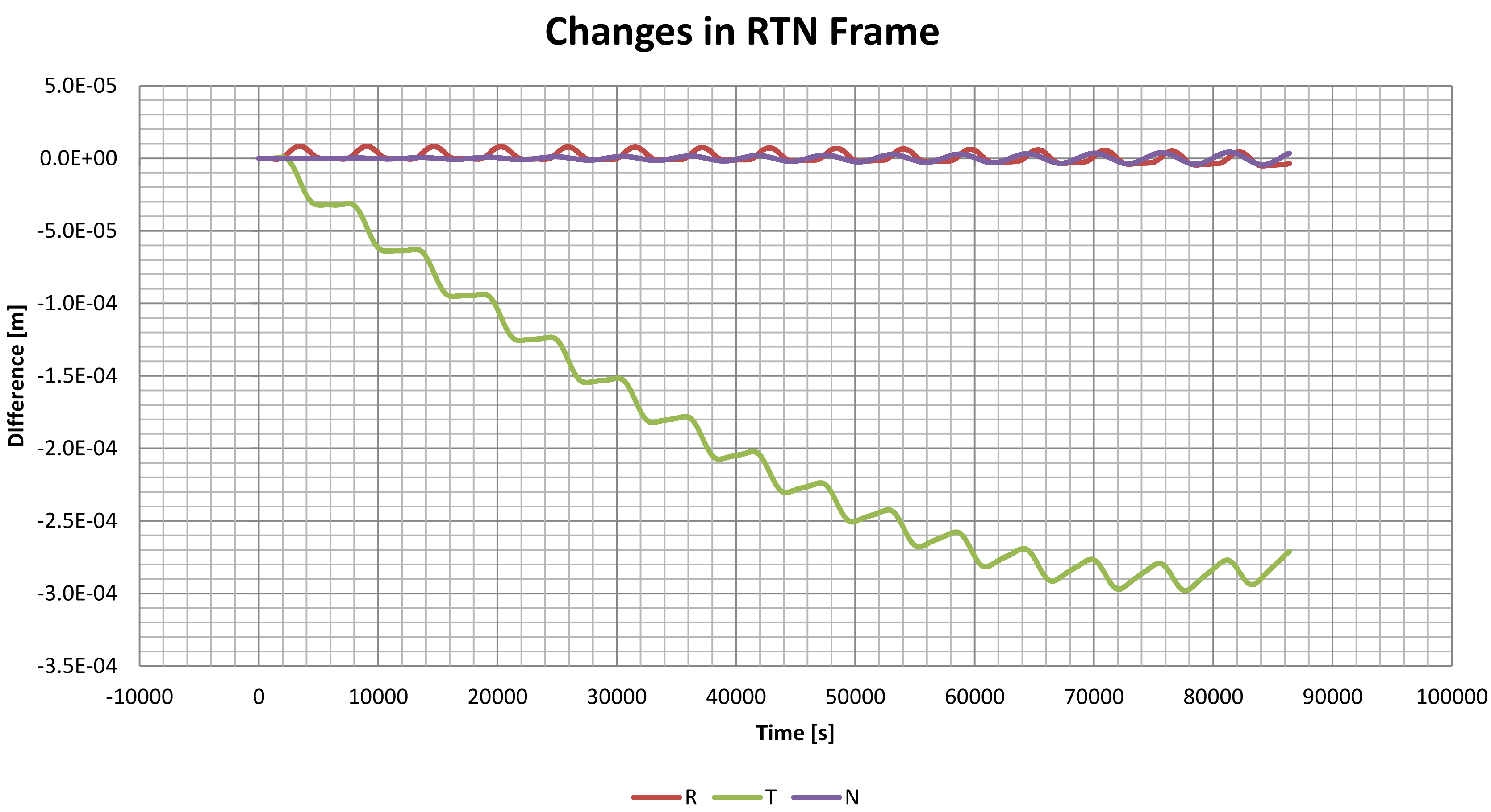
Results: ACC vs. Analytical



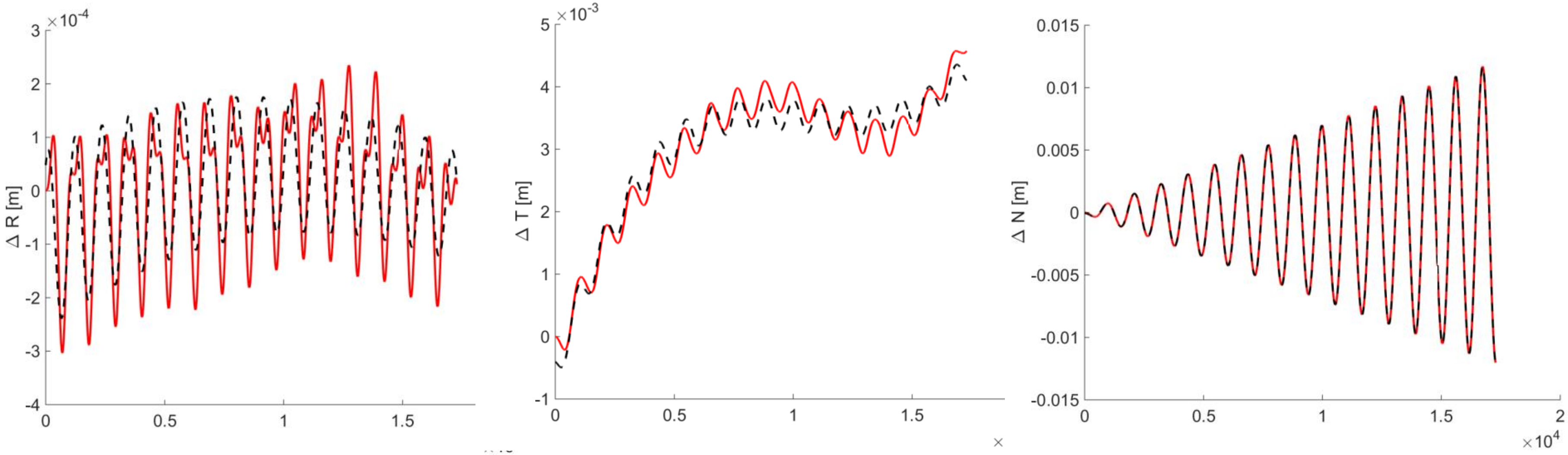
Case 1: Y is radial direction offset ACC+Y vs. ACC only



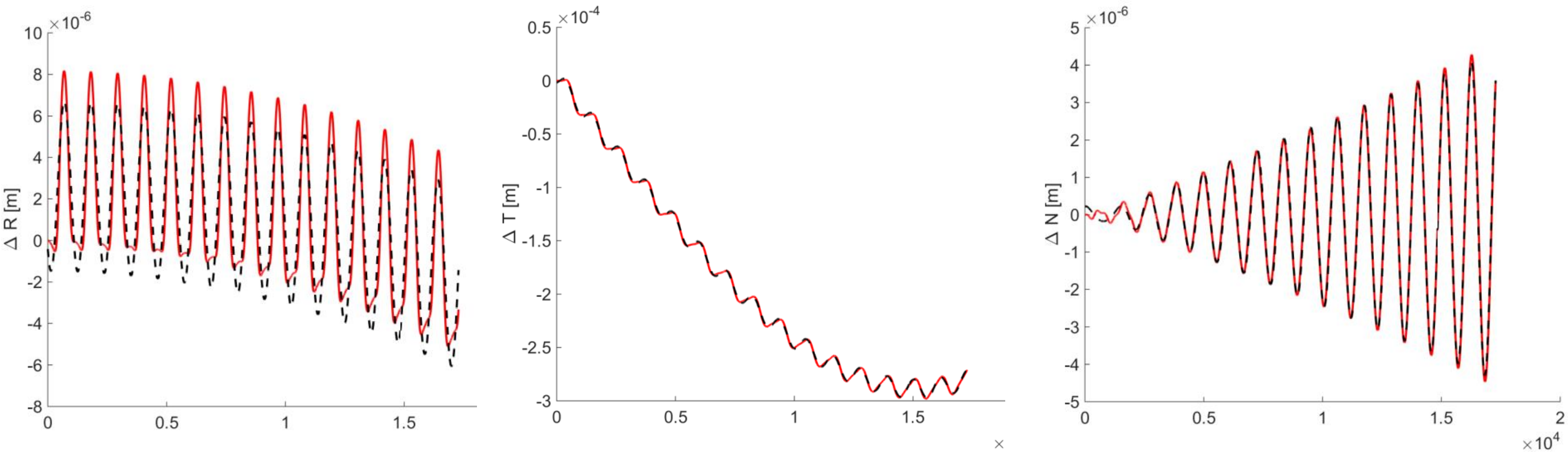
Case 1: Y is radial direction offset
ACC+Y+J2 vs. ACC+Y



Case 1: Y is radial direction offset



ACC+Y vs. ACC



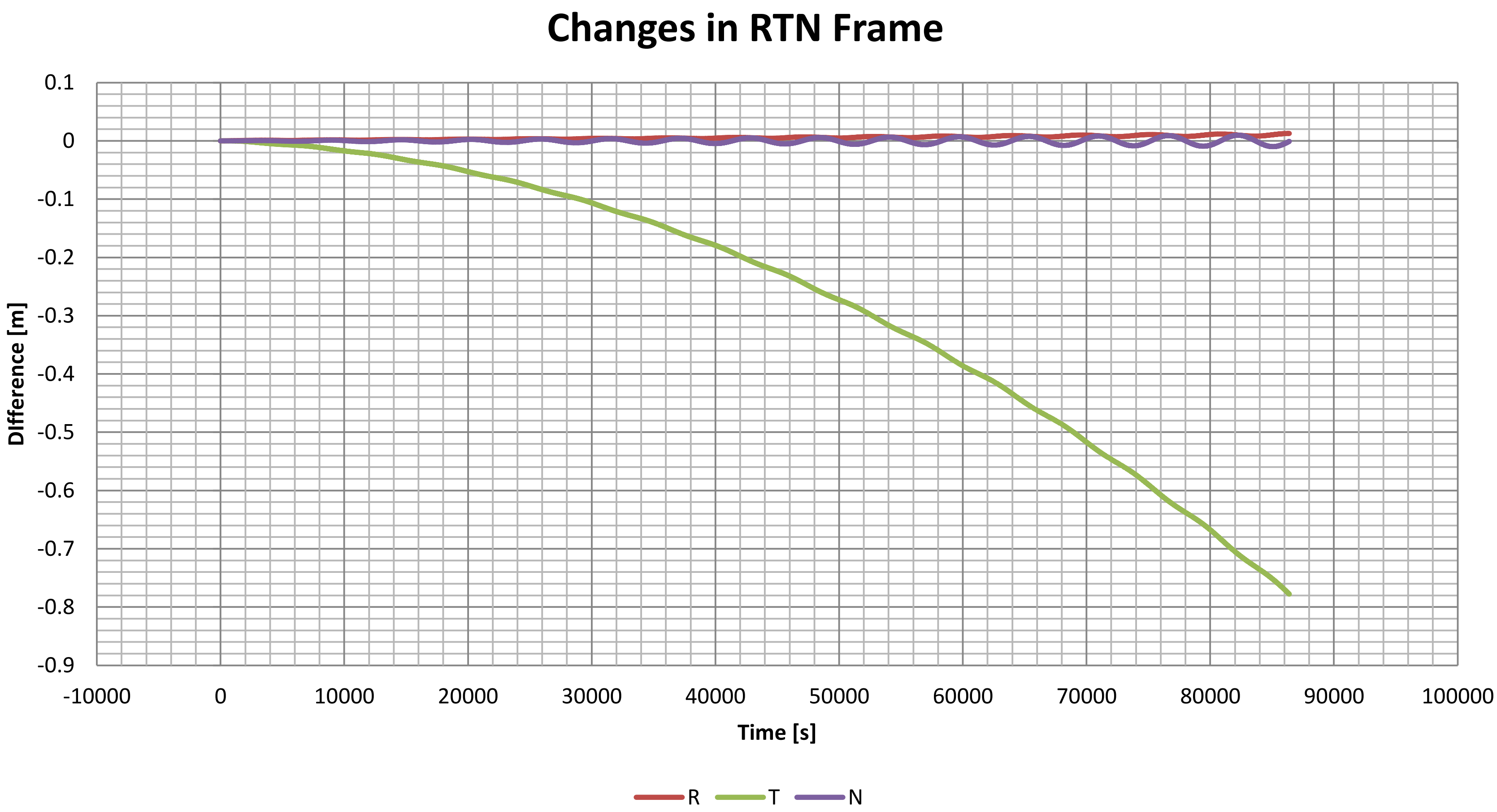
ACC+Y+J2 vs. ACC+Y

Least Squares Fits

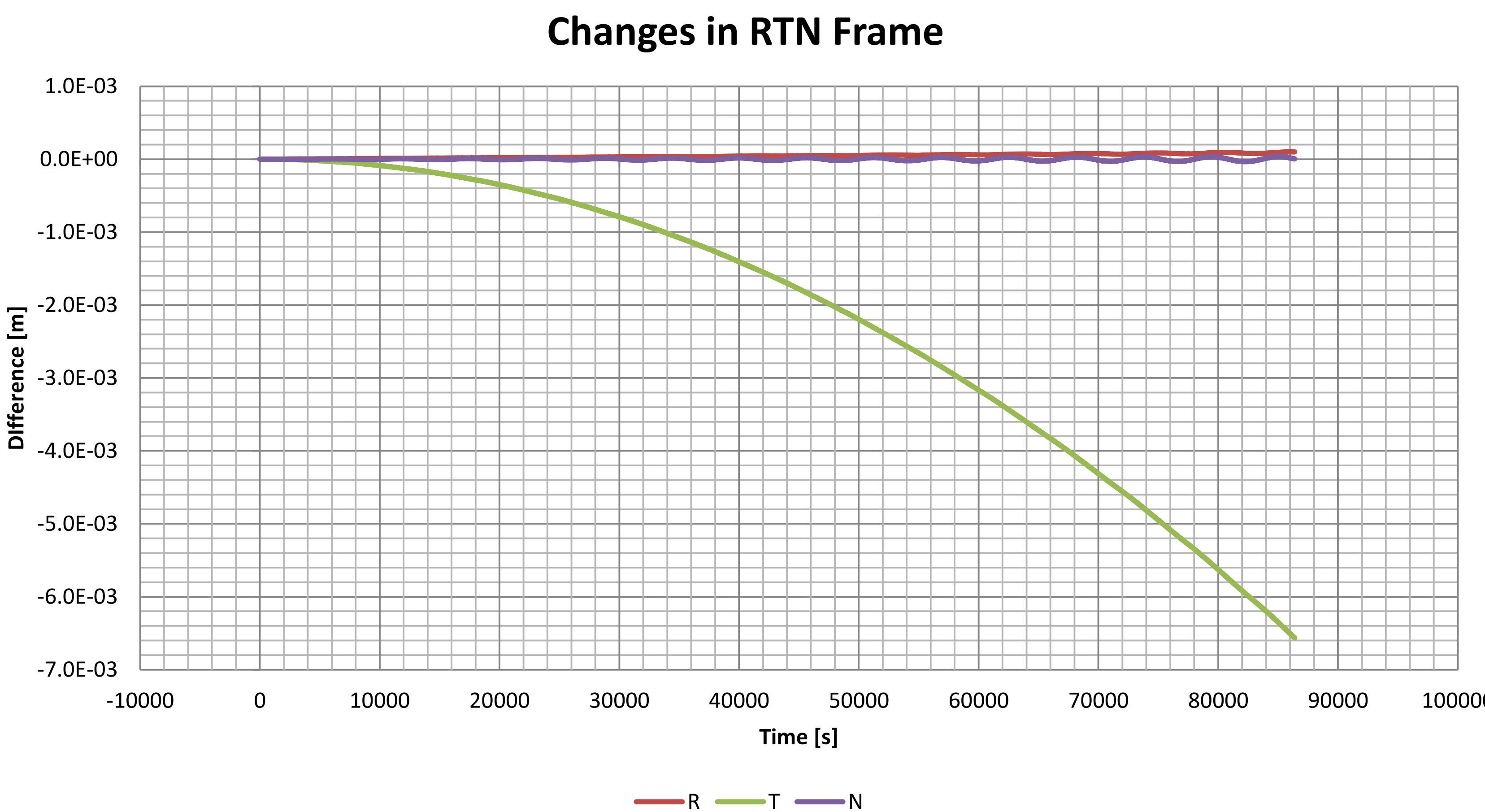
	Y – radial					
	ACCY – ACC			ACCYJ2 – ACCY		
	R	T	N	R	T	N
a	-9.29E-05	-0.00057	8.51E-06	2.61E-06	3.77E-06	-2.41E-08
b	5.68E-09	2.45E-07	-6.63E-10	-3.60E-13	-5.68E-09	1.85E-12
c	-5.77E-14	-4.74E-12	8.35E-15	-5.94E-16	-2.36E-15	-2.31E-17
c'		2.96E-17			3.48E-19	
d	0.00017	-3.15E-05	-0.00011	-4.08E-06	-6.21E-07	2.42E-07
e	-8.33E-10	1.83E-09	9.57E-10	-2.57E-12	1.61E-11	-2.81E-11
f	-1.52E-05	-0.00031	-6.35E-05	-3.15E-07	8.30E-06	-9.27E-08
g	6.66E-10	1.20E-09	-1.39E-07	1.25E-11	5.48E-12	4.51E-11
R^2	0.701523	0.967445	0.999972	0.894963	0.999809	0.996434

$$\hat{r} = a + b\Delta t + c\Delta t^2 + c'\Delta t^3 + (d + e\Delta t)\cos(\omega + \nu) + (f + g\Delta t)\sin(\omega + \nu)$$

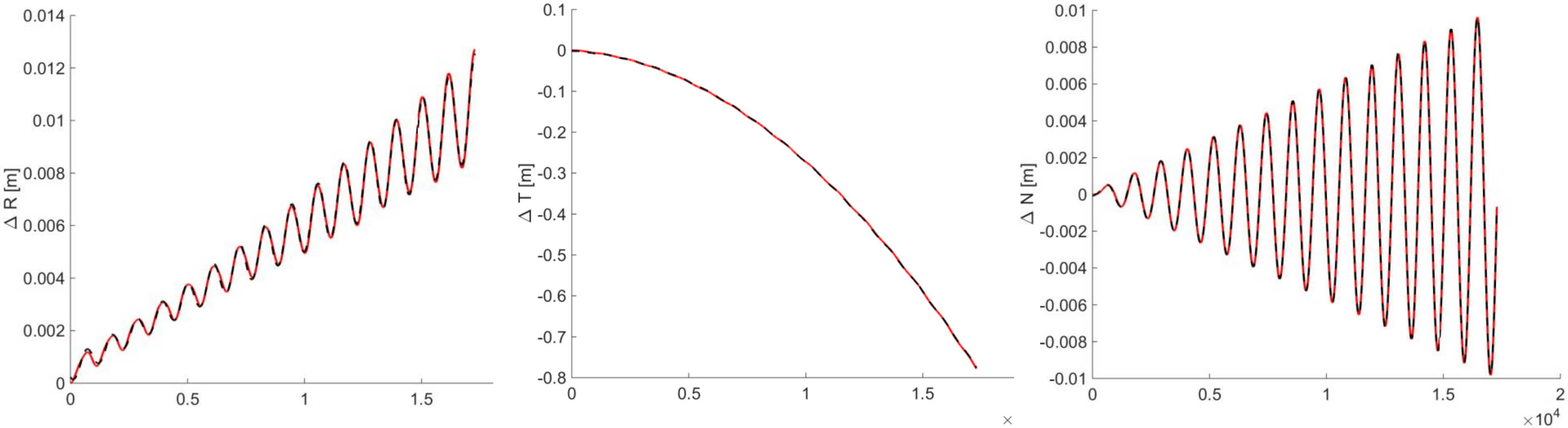
Case 2: Y is along-track direction offset ACC+Y vs. ACC only



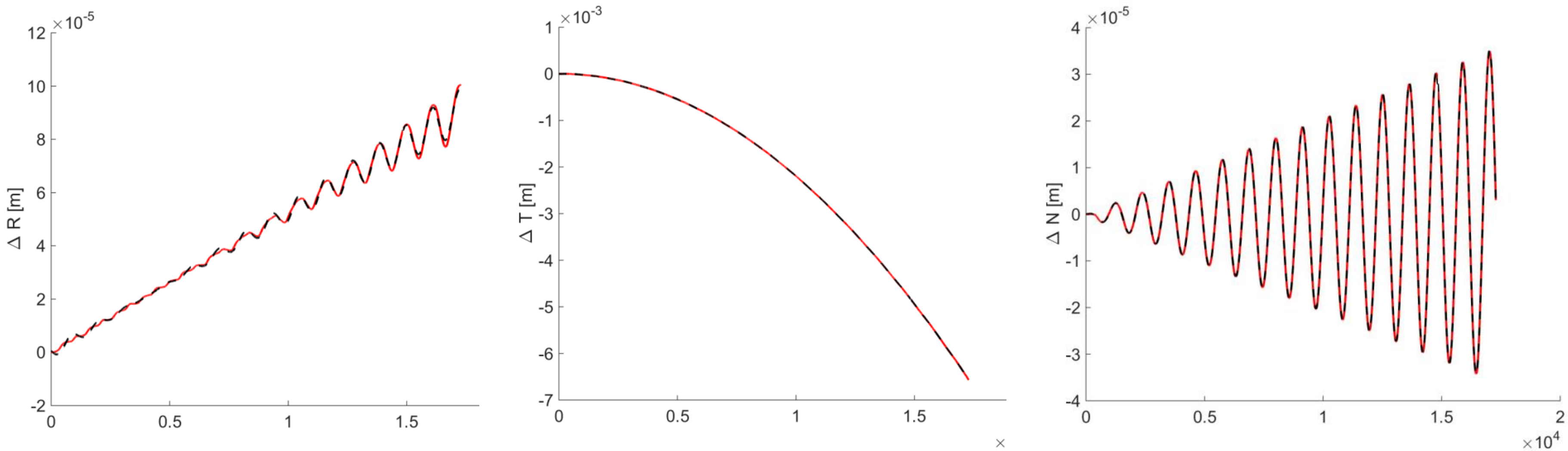
Case 2: Y is along-track direction offset ACC+Y+J2 vs. ACC+Y



Case 2: Y is along-track direction offset



ACC+Y vs. ACC



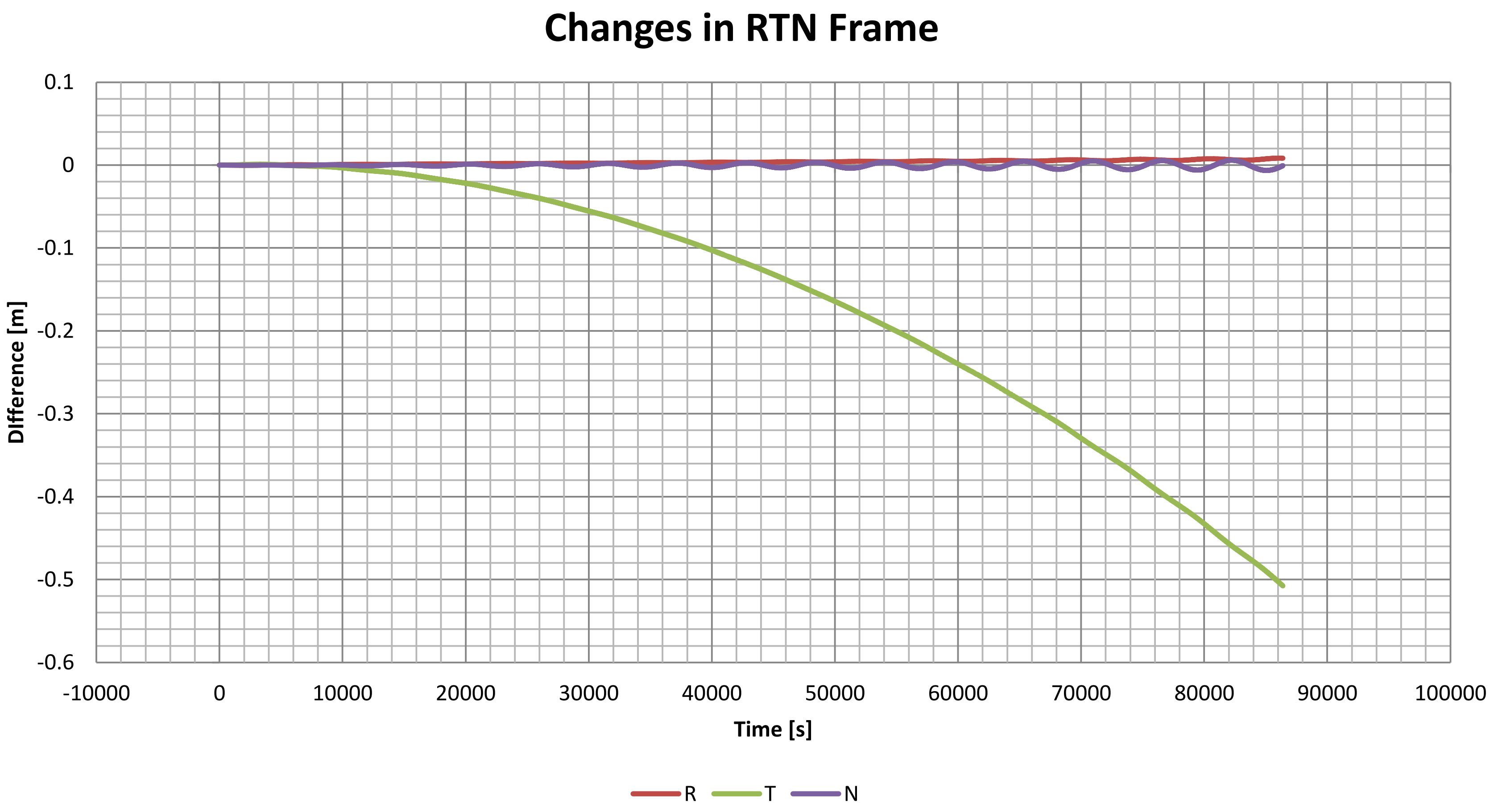
ACC+Y+J2 vs. ACC+Y

Least Squares Fits

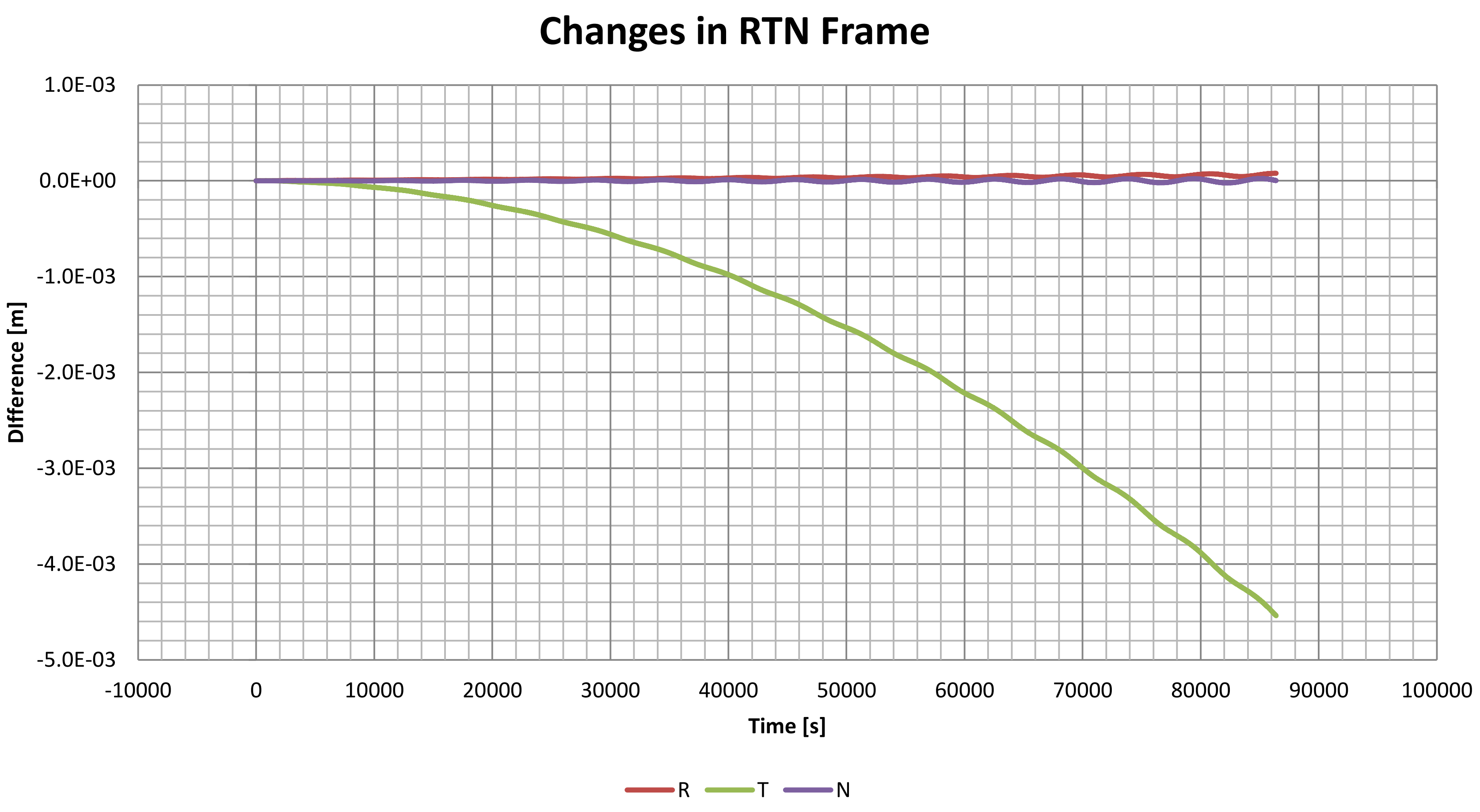
	Y – along-track					
	ACCY – ACC			ACCYJ2 – ACCY		
	R	T	N	R	T	N
a	5.09E-04	-0.00127	8.40E-05	1.30E-07	1.51E-06	-1.92E-07
b	1.06E-07	-5.79E-07	3.73E-10	1.04E-09	3.69E-11	7.13E-13
c	1.23E-13	-9.70E-11	-5.38E-15	6.03E-17	-8.80E-13	-3.25E-18
d	-0.00043	0.000241	-3.23E-05	-1.17E-06	-2.17E-06	6.44E-08
e	5.97E-09	2.39E-08	-1.16E-07	6.07E-11	5.60E-11	4.13E-10
f	-1.10E-04	0.000728	2.32E-04	-2.26E-06	1.65E-06	-2.28E-07
g	2.41E-08	-7.33E-09	-2.34E-09	1.08E-10	-6.78E-11	5.92E-12
R^2	0.999316	0.999983	0.999941	0.999016	1	0.999978

$$\hat{r} = a + b\Delta t + c\Delta t^2 + (d + e\Delta t)\cos(\omega + \nu) + (f + g\Delta t)\sin(\omega + \nu)$$

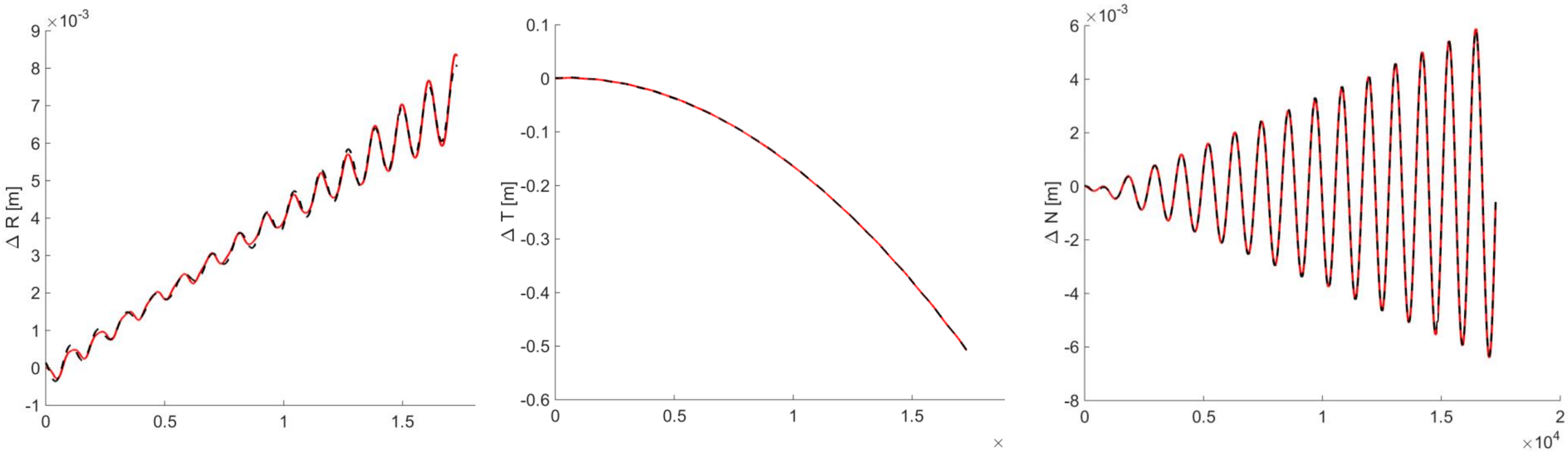
Case 3: Y is cross-track direction offset ACC+Y vs. ACC only



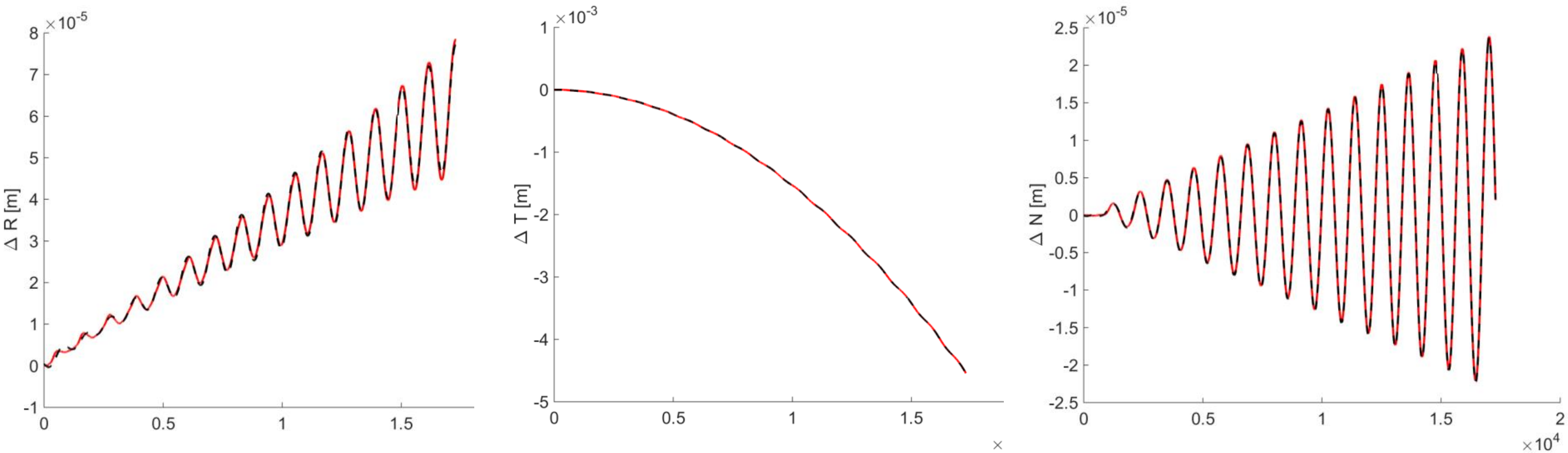
Case 3: Y is cross-track direction offset ACC+Y+J2 vs. ACC+Y



Case 3: Y is cross-track direction offset



ACC+Y vs. ACC



ACC+Y+J2 vs. ACC+Y

Least Squares Fits

	Y – cross-track					
	ACCY – ACC			ACCYJ2 – ACCY		
	R	T	N	R	T	N
a	-0.00017	0.001069	-0.00014	5.91E-07	5.98E-07	2.82E-07
b	8.54E-08	2.44E-07	6.03E-11	7.09E-10	-6.78E-10	-1.20E-13
c	-2.94E-15	-7.09E-11	-9.51E-16	8.47E-17	-6.00E-13	4.02E-18
d	9.22E-05	-0.0006	0.000181	-1.26E-06	-1.21E-06	-5.82E-07
e	3.49E-09	1.58E-08	-7.50E-08	4.03E-11	2.58E-10	2.81E-10
f	-0.00039	-9.10E-05	-3.38E-05	-1.45E-06	1.81E-06	-2.33E-07
g	1.41E-08	-2.04E-09	1.40E-09	1.86E-10	-4.44E-11	2.65E-12
R^2	0.998638	0.999996	0.999878	0.999102	1	0.999849

$$\hat{r} = a + b\Delta t + c\Delta t^2 + (d + e\Delta t)\cos(\omega + \nu) + (f + g\Delta t)\sin(\omega + \nu)$$

Conclusions (for 1 day)

- Integrated elliptical orbit have large disagreement with GNV1B trajectory (starting at oscillating/nonexact IC)
- ACC readings yields average altitude drift of $-0.5m$
- Radial Y gives a disturbance of $< [.03\ 0.5\ 1.5]cm$ in RTN
- Along-track Y gives $[1.4\ 80\ 0.8]cm$ in RTN
- Cross-track Y gives $[0.9\ 50\ 0.6]cm$ in RTN
- Considering J2 potential produce additional differences of $[8\ 300\ 5]\mu\mathrm{m}, [100\ 6500\ 35]\mu\mathrm{m}, [80\ 4500\ 25]\mu\mathrm{m}$ respectively in RTN
- Results should be free of numerical artifacts as difference operations have error $< 10^{-8}m$, integration $< 10^{-10}m$